

For office use only	Team Control Number	For office use only
T1 _____	8201	F1 _____
T2 _____		F2 _____
T3 _____	Problem Chosen	F3 _____
T4 _____	B	F4 _____

2017
HiMCM
Summary Sheet

Our team was tasked with designing a ski resort set to be built over the Wasatch Peaks Ranch property. Specifically, the resort had to have plenty of trails, main slopes of varying lengths, at least 160 total kilometers of slopes and trails, and a slope distribution approximating 20% green circles, 40% blue squares, and 40% black diamonds. We obtained GIS data of the resort location and used gradient vector fields and Mathematica’s morphological image processing capabilities to mathematically determine the locations of “fall lines,” the safest and most optimal ski slopes on the property. The robustness and stability of our model was determined by running an agent-based watershed analysis within SURFER to simulate fall lines. Through visual inspection of the few flat regions, we added in ski lifts and ski lift stations, marking where the slopes would start and end. Overlaying these three maps on top of each other allowed us to identify the most ideal paths: ones following the slope. As did many Olympic venues before ours, we want to showcase the beauty of the American landscape for the world by making as little alterations to the natural landscape as possible: all model and ski slope designs were built using existing fall lines and around landforms to provide for an eco-friendly and cost-effective ski slope.

We tested our design by comparing it to other selected resorts in North America. Our metric took into account three main factors: land per lift, slopes per lift, and slope difficulty distribution. By comparing them to optimal values, the metric produces a rating for each ski resort. We also created a computational agent-based model in NetLogo which allowed us to manipulate variables and test responses of our resort to stochastic variations in skier speed and lift distribution.

Our final solution consists of a total of 199.4 km of path, with a distribution of 22.72% green circle, 38.50% blue square, and 38.77% black diamond. The metric ranks Wasatch Peaks Ranch fourth among the sampled North American resorts. Wasatch Peaks has potential to become a highly desirable ski resort, as well as a strong contender to host of the 2026 Olympics.

Dear Ms. Mogul,

Thank you for allowing us to contribute to the future of the Wasatch Peaks Ranch and for the opportunity to design what may become a future Olympic host, or at the very least, one of the top ski resorts in all of North America. We are extremely excited to have this opportunity, and hope that you will take our suggestions into consideration.

After critical analysis of the problem, we used QGIS software to model the property and its surrounding land as a colored image in which the color of each pixel was determined by its grade and which of the following ranges it lied in: 0-6%, 6-25%, 25-40%, and 40-100%. Upon obtaining this, we superimposed the outline of the property onto this image so that we could see the difficulty of a slope based on the pixels it has crossed through. Here we identified ski lifts and ski lift stations (Figure 7). After, we combined this image with a gradient field to identify possible slopes and separated these by difficulty. This yielded 45.31 kilometers of green circle trails, 76.77 kilometers of blue square trails, and 77.32 kilometers of black diamond trails, which gives us a distribution of 22.72-38.5-38.77 for beginner, intermediate, and advanced slopes. In total these 254 trails of varying lengths span 199.4 kilometers, thus satisfying all of your requirements.

Once we had a design, we analyzed certain characteristics of ski resorts in general. We decided that the casual skier valued slopes being in walking distance of ski lifts, not having to wait long periods of time after getting off of lifts, and a good distribution of trails, which we decided to be 20 - 40 - 40 as per your suggestion. We developed a metric which gave scores for each of these characteristics based on percent difference between what the resort had and what we determined to be optimal values. The score, more specifically, was the difference between the percent difference and 1. The total rating was determined by multiplying these scores together and multiplying that value by 100. This metric is very tough and as a result, in general ratings are low and sometimes even negative. There's no need to worry; our design received a rating of 44.7, which ranks fourth among a selection of North American ski resorts, behind Vail (57.6), Jackson Hole (49.0), and Silver Star (48.6).

In order to model changing characteristics such as lifts and skiers, we developed an agent-based model in NetLogo which took in a series of inputs such as lift length and speed, and returned values for lift wait time and the amount of skiers on a slope at a given moment. The model showed us that for Wasatch Peaks Ranch, we would be best served by having 26 ski lifts active during peak hours. Additionally, the model distributes the lifts according to the trail distribution, 20 - 40 - 40. This indicated that we needed 5 lifts servicing beginner green circle trails, 10 servicing intermediate blue square trails, and 11 servicing advanced black diamond trails.

Using topological data provided by the US Geological Survey, we constructed maps which took into account the grade of the land, as mentioned earlier, the solar aspect, and the fall lines. The solar aspect maps the location of the sun in relation to slopes and ridges. Slopes facing north were colored red, and slopes facing south were colored blue. This allowed us to identify and isolate those slopes which occurred on northern edges. Solar radiation on the southern edge leads to melting and refreezing - and icy conditions poor for skiing. Maps of fall lines were also created in order to identify the path of steepest slope down a ridge. These paths travel perpendicular to the contour lines, and are oftentimes easier to maneuver. When traveling along a fall line, acceleration due to gravity pulls the skier downwards on the path. On the other hand, when traveling along the contour lines, acceleration due to gravity will pull the skier off the path.

Combining these three maps allows us to not only identify all possible ski paths, but to narrow these paths down to only the most ideal ones - ones that aren't too steep or shallow, travel in the direction of the slope, and are located on northern faces.

Lastly, we hope you will be inclined to choose our design (Figure 9), because its implementation is very environmentally friendly. As you may already know, the property is consistent of many creeks and rivers. None of our trails cross bodies of water, meaning that we will not need to disturb them. Because, we do not need to alter the environment much, we can expect our design to be highly cost-effective and attractive to tourists as it preserves the national landscape for all to behold.

Sincerely,
Team 8201

Enclosure
Model

Introduction and Restatement of Problem

The Winter Olympic Games are a major international event, drawing athletes and crowds from over a hundred different countries. A developer is seeking to turn Wasatch Peaks Ranch in Peterson, UT into a top ski resort as well as potential host location for the Winter Olympics. The nearly 13,000 acre location contains an potential estimated 5,500 acres of ski slopes, an 11 mile ridgeline, and a 4750 foot elevation drop among its 24 peaks and 15 bowls. The goal of our paper is to identify potential ski slopes and trails, as well as rank the resort against existing North American ski slopes.

Assumptions and Justifications

Certain assumptions were made in our solution in order to reduce the number of variables to be taken into account.

Assumption 1: Ski slopes must be placed on ridges primarily, but they can be placed in valleys for beginner courses.

Justification: Skiers are only safe if they are sliding *down* a slope; an incline *across* a mountain would cause them to drift to the side, into a valley. The only paths down a mountain (shown in the figure at right) with no cross-slopes are down the valleys (blue) and ridges (red). This path follows the fall line, perpendicular to the contour lines on a topographic map.

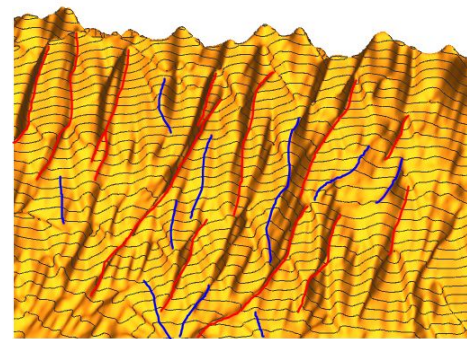


Figure 1: Mountain with valleys highlighted in blue and ridges highlighted in red

Assumption 2: USGS data from 2008 is representative of topographical data for the Davis and Morgan County areas today, and will continue to be until at least the 2026 Winter Olympics.

Justification: Using the online USGS topoView tool, we overlaid the oldest reliable online data, coming from a 1954 survey, onto the 2008 dataset, and found no significant alterations caused by erosion or climate change between the two. Elevation values and landforms were reasonably similar so that we could assume that they wouldn't change until 2026.

Assumption 3: The outparcel lot cannot contain any ski slopes.

Justification: Outparcels are areas of land zoned for a specific type of development, which in this case is the resort. As we don't know the developers' plans for the outparcel, we will not put ski slopes anywhere within its bounds.

Assumption 4: A person will start skiing 10 seconds after the person who starts before him/her.

Justification: We decided that 10 seconds was a safe following time, because it allows the person in front of you to establish an appropriate distance between the two of you. In addition, our NetLogo model supported this idea, demonstrating a substantial increase in safety.

Assumption 5: The number of skiers of each experience level is proportional to amount of trails of certain difficulty level. Skiers are also distributed evenly among the lifts serving their experience zones.

Justification: We have no data on the distribution of skill levels among skiers to use. If the distribution of skill matches the distribution of trails, then all trail levels will have the same level of 'traffic,' because they will have the same trail-to-skier ratio and all skiers will be at the same skill level. An even distribution is the quickest way to remove congestion, because someone goes down the hill via each slope every ten seconds and if the amount of people waiting for each slope is equal, then the last person in each line will go down the hill at the same time.

Assumption 6: Ski slopes and lifts will not be placed on disconnected lots.

Justification: On the topographic map of the property we see that there are three small areas which are disconnected from the main body. It would not make sense to include any slopes that begin or end on these properties, since there is no access route that does not trespass onto neighboring lands. Additionally, their areas are so small, roughly 20 acres, that it would not make sense to have a ski run begin and end on the property.

Assumption 7: The cost of building a slope anywhere within the ranch is the same.

Justification: There is no monetary advantage to building slopes in one place versus another and thus we should not try to include more slopes in certain regions for this reason.

Assumption 8: Ski slopes cannot intersect creeks.

Justification: Creeks make slopes more dangerous, by creating thin slippery ice, which is unsafe to ski over. It is important for us to take this into account, due to the significant amount of creeks winding through the property. Oftentimes, mountain streams and

Assumption 9: On average, people will walk 0.5 km at most to where they begin skiing.

Justification: According to a study published in the American Journal of Preventative Medicine on walking distances, the most a person is willing to walk for recreation is 0.76 kilometers [11]. However, they are likely to walk 0.13 kilometers less in the winter time. However, this is not accounting for location; this data was collected from people walking in cities and suburbs. We expect that in mountainous terrain, the amount they are willing to walk will decrease by twice as much, to 0.5 kilometers.

Assumption 10: The average length of a ski run is 1 kilometer and the average width is 10 meters.

Justification: From previous data, we were able to extract the average length. We found the amount of slopes in each resort and determined the average length of ski runs in each resort. The mode of this data was 1 kilometer, which appeared for 7 out of 16 resorts. In addition, to verify the validity of this assumption, after ranking the resorts, we took a weighted average of the average lengths in the resorts that ranked above the 50th percentile and that yielded an average length of 0.99 kilometers. The average width of a run was decided by us as 10 meters, simply so that they are not too narrow.

Assumption 11: Constant numbers of people on slope throughout day.

Justification: Time is the main remaining factor influencing congestion in the resort. If time does not influence congestion, then we can expect that to be constant at all times throughout the day, making it simpler to model.

Assumption 12: Ski lifts must begin and end on flat areas of land.

Justification: Limiting ski lifts to flat areas means that they can only be placed on shallow slopes, which we determine to be beginner areas highlighted in green or in flat areas highlighted in red. Ski lifts should not be placed on any steep slopes, in order to allow for riders to properly stop and mount their gear once they egress the lift area.

Model Design

In order to find the optimal ski slopes on the ranch, we first obtained Digital Elevation Model files (.dem) of the area near Peterson, UT from the U.S. Geological Survey's National Mapping

Program [10]. This was much higher resolution than the dataset provided within the problem statement and the Wasatch Peaks Ranch website (7.5 arc minutes compared to 30 arc minutes) and enabled us to analyze the topography of the area with much more precision than by using the topographical map provided. The USGS data is from 2008, but we assumed this dataset was representative of current topography of the Wasatch area and the area’s topography in the future, as we are planning for the 2026 Winter Olympics. This has been elaborated on in the Assumptions section. The data was imported into Mathematica and graphed in three dimensions, obtaining the topographic figure shown below. The data that we found for Peterson county omitted the far northern and southern tips of the trail, so we also obtained elevation files for the cities north and south of Peterson, Bountiful Peak and Snow Basin. Unfortunately, the curvature of the earth causes slight inaccuracies in these satellite data files, so there was a visible “seam” between the counties, which is visible in later figures.

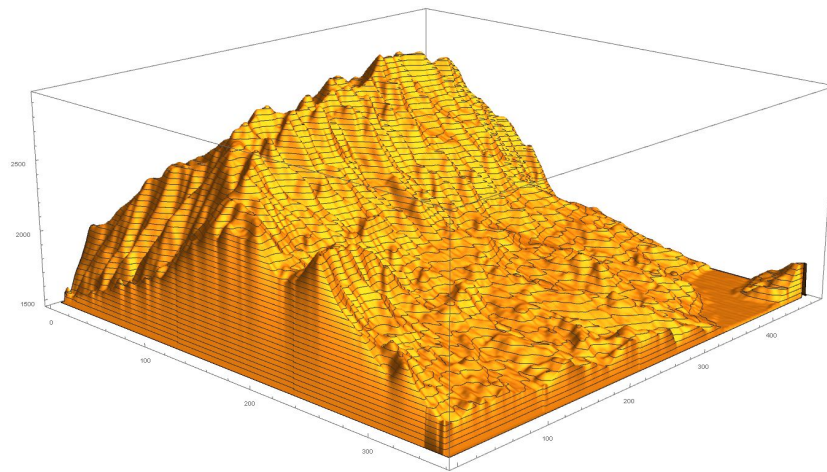


Figure 2: 3D topographical graph of Peterson, UT, elevation exaggerated

Once we obtained this higher-resolution dataset of the elevation (332 data points by 457 data points), we interpolated the gradient vector of every point based on the slopes in the directions of the nearest 8 other points. This created a vector field in which every vector in the direction of the steepest angle around it with magnitudes corresponding to this steepness, using the equations below. This steepest angle is the “slope” of a mountain according to the terminology used by skiers.

$$\frac{(A+2D+G)-(C+2F+I)}{8*cell\ size} = Z_x^2$$

$$\frac{(A+2B+C)-(G+2H+I)}{8*cell\ size} = Z_y^2$$

$$\% slope = \sqrt{Z_x^2 + Z_y^2}$$

A	B	C
D	E*	F
G	H	I

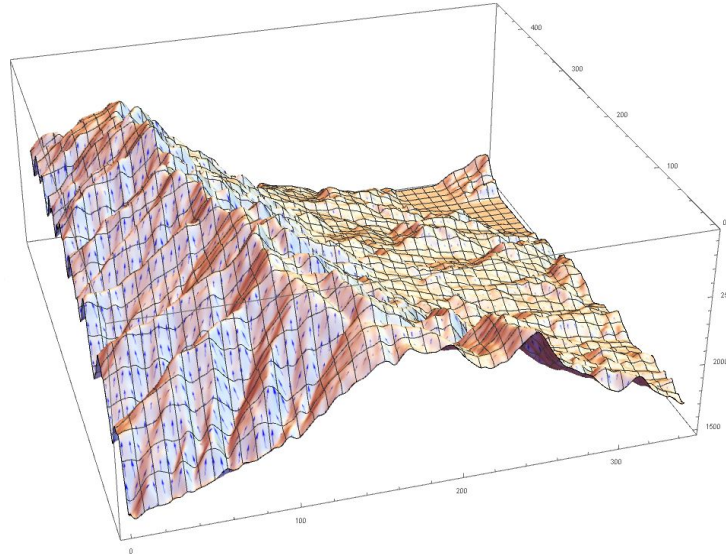


Figure 3: 3D wireframe of Peterson, UT, with gradient vectors for every point overlain

We used this vector field to find the “ridges” and “valleys” in the ranch, as these are areas of the mountain with no cross-slope, making skiing safest in this area. We did this by converting the direction of every gradient to an angle between $-\pi$ and π . After this, we displayed the data as a grayscale image such that darker gray corresponded to $-\pi$, and used Mathematica’s state-of-the-art image analysis capabilities to find the edges (high contrasts between light and dark) of our vector field. These edges represented the areas in which vectors point most directly at each other, defining mathematically all ridges and valleys in the image.



Figure 4: Close up of colorized ridges and valleys in the ranch. Each pixel in this image represents a distance of 30 meters. Note the seam caused by GIS stitching in the center of the image.

This was our base representation of all possible ski slopes in the area around the ranch. We found that there were 1,235 kilometers of possible skiing paths in the area around Peterson. This was done by the conservative estimate that one colored pixel, which represents a square with side length 30.01 m, also represents 30 meters of trail length, though a longer trail could conceivably fit within a 30-meter square. The Coastline Paradox states that since a landmass has features at all scales, kilometres in size to millimeters and below, there is no definable size of the smallest feature that one should measure around, and therefore no well-defined perimeter to the landmass. A similar problem arises when measuring other topographical features such as ridges; it's not possible to obtain a concrete value for trail length in a region, so we decided this metric was valid, considering our lack of comparison points.

To filter this excessive path length, we eliminated small connected sets of ridges, as many of them were caused by errors with our gradient interpolation, or they represented regions too small to ski down. We determined the connected sets of ridges using Mathematica's `MorphologicalComponents[]` command, which was able to analyze the size of each contiguous white-pixel region in our binary image file. We colored these components in Figure 4, and then used the `DeleteSmallComponents[]` command to delete regions less than a certain size in pixels. We settled on 40 pixels as a minimum, as this was the smallest number able to remove all circle-shaped morphological components around tiny un-skiable peaks.



Figure 5: All ridges and valleys in Wasatch Peaks Ranch

We then wanted to determine the grades of these various slopes, to determine which fell in which categories of slope. Using QGIS, an open source Graphic Informational System, we created 2D plots with the same vector fields as earlier for each individual pixel in our three elevation files, and we stitched these files together to create one image, mapping all of the ranch.

Grades are calculated as the *percent slope*, as opposed to the degree angle, such that a 100% grade occurs when the angle is 45 deg. From here, we colored the image so that red represented grades less than 6% and greater than 100%, grades which cannot be skied upon. We noted that the highest grade was approximately 75%. Of the remaining skiable acreage, we colored pixels with grades between 6 and 25 percent green to show which pixels could be part of slopes, pixels with grades between 25 and 40 percent blue to show which pixels could be part of blue square and black diamond slopes, and pixels with grades between 40 and 100 percent black to show which pixels could be part of black diamond slopes. After obtaining this image, we superimposed an outline of the ranch onto the stitched colored image. The figure is shown on the following page in Figure 6.

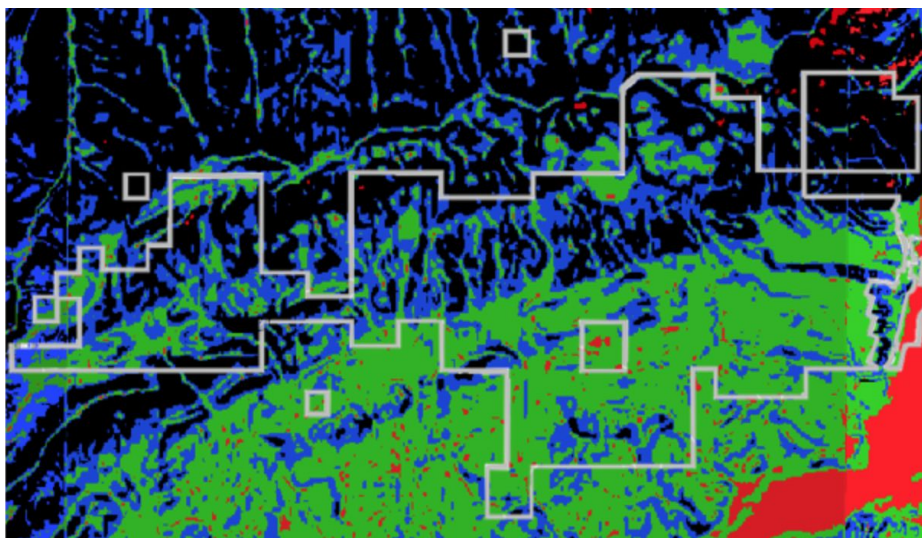


Figure 6: Slope Gradient Map with Wasatch Peaks Ranch overlaid in grey.

Next, we added in features of resorts such as ski lift stations and where ski paths could start and end through visual inspection. It made sense to us that those aforementioned places could be placed over red pixels, because those areas are too flat for skiing. (Note: Since the maximum grade in our data was 74.5%, none of those pixels are too steep.) We decided upon thirteen main areas for starts and ends of trails, which we made ski lift stations.

Based on this, we added in twenty-six ski lifts. With these lifts, the longest ski lift trip between two stations is four rides. For most stations, you have the option to go either east or west to the station at the closest latitude and its longitudinal adjacencies as well as its own longitudinal adjacencies. This is shown below in Figure 7.

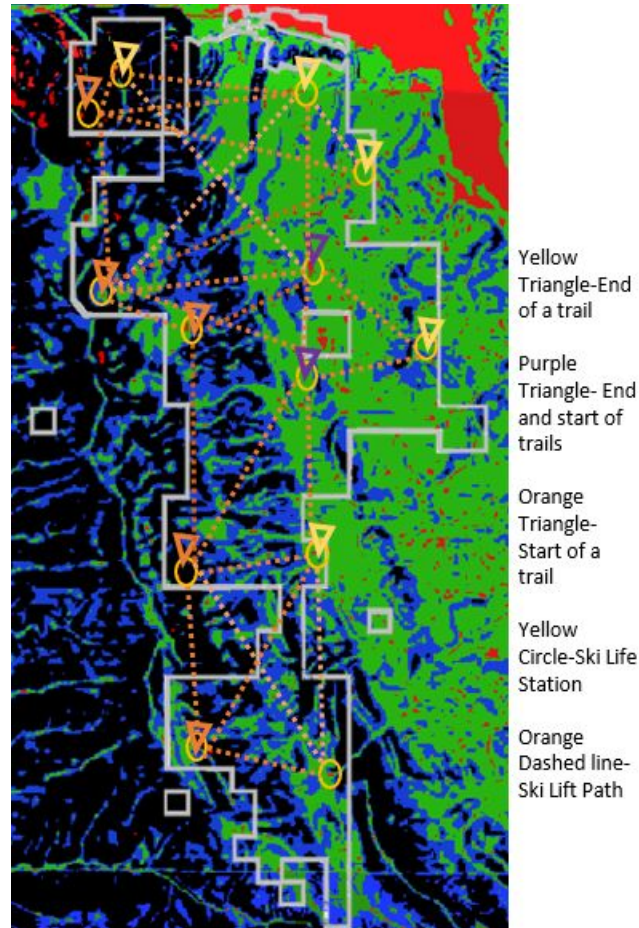


Figure 7: The Map with Resort Features

Next, we calculated a vector field of fall lines using topological and altitude data. Fall lines occur perpendicular to the contour lines on a topographic map, and follow the path of steepest slope down a ridge. In the case of our ski resort, paths that travel along a fall line are easier to travel, as acceleration will push a skier along the path. On the other hand, paths that cross fall lines and travel parallel to contour lines are harder to follow since the force of gravity pulling on the skier will be pointed perpendicularly to their path of motion. The vector field containing these fall lines is calculated using QGIS. When these fall lines are overlaid on the slope gradient, we can clearly see what difficulty each potential path would be categorized as: either green circle, blue square, or black diamond. Furthermore, stacking these paths over the solar aspect map allows us to further whittle down the number of potential paths by focusing on ones that rest on the north face of a ridge.

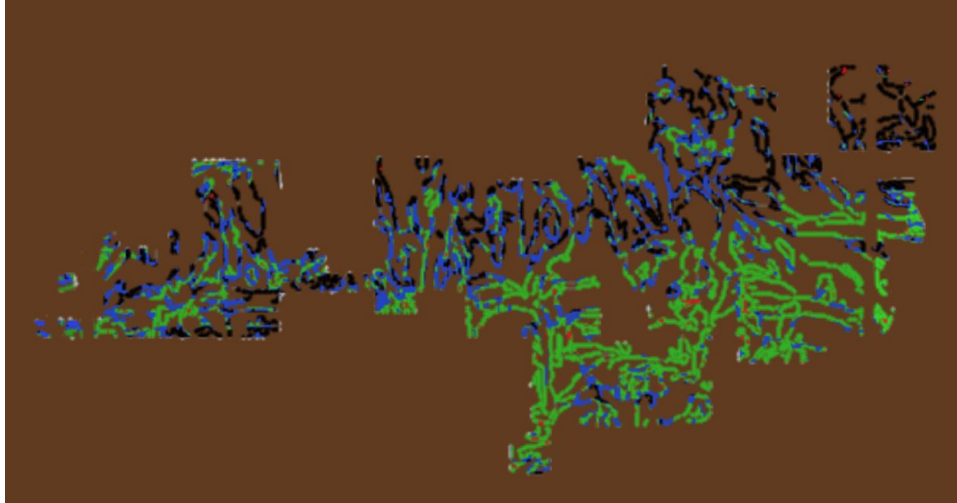


Figure 8: Fall Lines Masked over the Slope Gradient. Each path on the image represents a potential trail.

For a final filtering of fall lines, we manually removed all trails which passed through one of the rivers on the property, as these are generally unsafe, if even possible, to ski over. We then sorted and recolored trails based on the steepest grade on their length, as trails are defined by their steepest gradient regardless of its total occurrence. Our solution diagram below combines all of our coding and manual editing, leaving only the most viable ski paths of diverse lengths and difficulties (green circle, blue square, black diamond). The resulting figure is overlaid on top of a street view image. In it, we can see a combined 199.4 km of slope. Of these 199.4 km, there are 95 green circle paths, 129 blue square paths, and 128 black diamond paths. There are 45.31 km of green path, 76.77 km of blue path and 77.32 km of black path. The total lengths of these paths form a ratio of 22.72% green circle, 38.50% blue square, and 38.77% black diamond. The sum of the percentage differences of each section from the target 20/40/40 ratio is 19.67%.

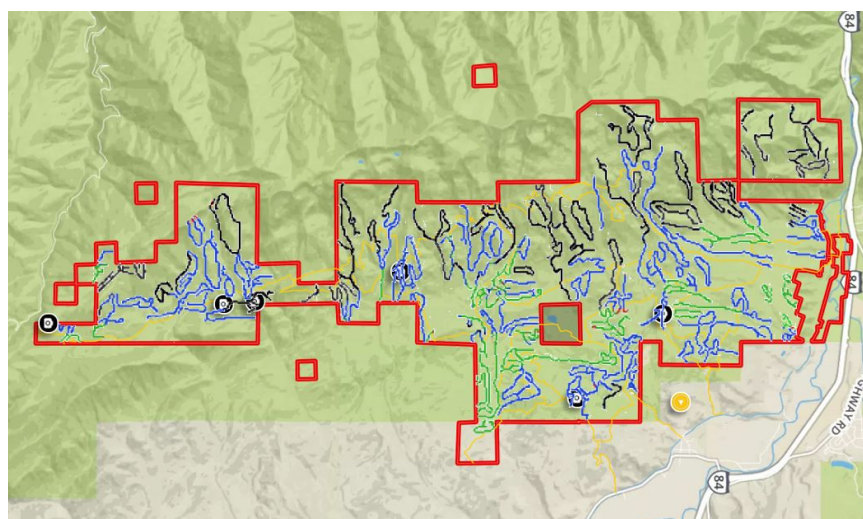


Figure 9: Map of ski slope locations overlaid over street map image. Color coded based on slope difficulty (green, blue, black).

Model Ranking/Testing

We devised a ranking metric in order to compare our model ski slopes with other ski slopes found in North America, including slopes used in the 2002 and 2010 Winter Olympics. The ranking metric takes several factors into account, including: how much walking space is split among lifts (WSPL), how many slopes are split among lifts (SPL), and the distribution of green circle (GCP), blue square (BSP), and black diamond trails (BDP) and compares them to what we found to be optimal values for each. All variables representing optimal values have the same notation, but are followed by a degree symbol (i.e. WSPL[°])

Our metric determined a rating out of 100 for each park through the following formula:

$$Resort\ Rating = 100 \times \left(1 - \frac{WSPL - WSPL^{\circ}}{WSPL^{\circ}}\right) \times \left(1 - \frac{SPL - SPL^{\circ}}{SPL^{\circ}}\right) \times \left(1 - \frac{|GCP - GCP^{\circ}|}{GCP^{\circ}}\right) \times \left(1 - \frac{|BSP - BSP^{\circ}|}{BSP^{\circ}}\right) \times \left(1 - \frac{|BDP - BDP^{\circ}|}{BDP^{\circ}}\right)$$

First, we determined the optimal values for each variable. We began with the walking space per lift. We defined the walking space to be the quantity of the land used for skiing subtracted from the total skiable acreage (TSA). Based on our assumption that all trails are ten meters wide, we found the area to be a hundredth of the total slope kilometers (TS) in square kilometers. We then multiplied that by the conversion to acres and subtracted it from the total skiable acreage, to come up with the formula:

$$WS = TSA - (2.47105381)(TS)$$

We decided that, in an “optimal setup,” every part of the walking space would be in walking distance (WD) from a lift’s end. Thus, we also derived the following expressions (where L is the number of lifts):

$$WS^{\circ} = \pi(WD)^2(L) \text{ and } WSPL^{\circ} = \pi(WD)^2$$

From research, we assumed that “within walking distance” was equivalent to within half a kilometer, so we substituted that value into the equation, converted to acres and found the optimal walking space per lift to be about 194 acres.

Next, we determined a value for the optimal number of slopes that each lift should take skiers to. We assumed that each skier would start a slope 10 seconds after the skier starting immediately

before him, meaning that if a ski lift carried people to the top of a hill where n slopes began, after 5 seconds, n people would leave the top of that hill. We agreed that the best resort would avoid congestion by having the same amount of people leaving as arriving.

With this in mind, we researched chairlifts and found that the best ski lifts can carry 4000 people to the top of a hill in one hour. From this, we gather that, about every 10 seconds, 11 people arrive at the top of the hill. This suggests that each hilltop should have 11 slopes covering approximately 11 kilometers as justified by our assumption that on average runs span 1 kilometer. However, since we do not have map information for each resort, we are judging the amount of slopes on a hill as the average number, calculated by the equation:

$$SPL = \frac{TS}{L}$$

Lastly, we are assuming that the distribution asked of us, is the distribution that attracts the most people. This assumption tells us that optimal green circle percentage is 20% and that the optimal blue square percentage and the optimal black diamond percentage are both 40%. These percentages have the following simple computations, based on the green circle slope kilometers (CS), the blue square slope kilometers (SS), the black diamond slope kilometers (DS):

$$GCP = 100\left(\frac{CS}{TS}\right), \quad BSP = 100\left(\frac{SS}{TS}\right), \quad \text{and} \quad BDP = 100\left(\frac{DS}{TS}\right)$$

From here, we calculated the ratings of some North American ski resorts. These ratings are shown in the table below.

Name	Resort Rating	Ranking
Beaver Creek	33.496	5
Big Sky Resort	35.585	4
Breckenridge	28.850	6
Fernie	16.410	9
Jackson Hole	49.027	2
Killington	7.137	14
Lake Louise	-8.103	16
Park City Mountain	9.976	12
Silver Star	48.574	3
Squaw Valley	13.425	10

Steamboat Springs	19.741	7
Sugarloaf Mountain	18.212	8
Sun Peaks	-3.624	15
Vail	57.634	1
Wasatch Peaks Ranch	44.764	4
Whistler Blackcomb	13.058	11
Winter Park Resort	8.081	13

Figure 10: Table of Resort Ratings and Rankings. Wasatch Peaks Ranch is highlighted.

In traditional rankings, Vail, Jackson Hole, Whistler Blackcomb, and Park City Mountain, consistently have high rankings. Our metric supports these rankings of Vail and Jackson Hole, but not of Whistler Blackcomb or of Park City Mountain. At first, this seems a little off putting, since both of these facilities were used as Olympic hosts in previous years. However, this is most likely the reason why they have such low ratings. For example, Whistler Blackcomb is very large and is full of difficult trails, which tend to be somewhat far from lift exits. The casual skier, who our metric appeals to, will most likely be deterred by this. Park City Mountain, on the other hand, doesn't have enough slopes. It is likely, that they focused on making really good slopes, and that they don't have enough slopes to clear congestion. That is not to say these resorts are bad, but merely that they will not be as appreciated by the average skier.

Computational Model

In order to gain a more in depth look at how individual variances would play a role in our model, we created an agent based model in NetLogo. The agent based model allows us to see minor variances which would otherwise have been obscured by averages in our mathematical model. The model we developed takes in several inputs, in the form of the number of people in the park at a given time, the speed of each lift, the capacity of each lift chair, the spacing between chairs, and the number of lifts in total. It outputs data in the form of two graphs, one of which shows the average wait time for a lift, and the other which shows the number of people presently skiing and riding the lifts at a given time.

The first part of the model sets up each skier. Each skier is randomly assigned to a skill level (beginner, intermediate, advanced) based on the target distribution of slopes - 20% beginner, 40% intermediate, and 40% advanced. A randomized component is incorporated in order to add variation and demonstrate how having percentages that differ from these goals affect the result.

Lifts are also distributed among the three common difficulty levels according to the 20 - 40 - 40 ratio.

Once the skiers and lifts are set up, queuing begins and the wait timer begins. The average time that a person waits before riding a lift is recorded and graphed. Once a skier boards the lift, they move at a constant speed, regardless of which lift it is. However, the lifts for the beginner paths are roughly 25% shorter than the lifts for intermediate and advanced paths. The reasoning being, that beginner slopes are flatter, and are located nearer to the outparcel lot, whereas the topographic map shows that steeper intermediate and advanced slopes are located further away.

At the top of the lift, skiers are let off and begin their path down. In addition to the randomized distribution of skiers, speeds are also weighted randomly. Each skill level has a corresponding base speed associated with it, but each individual skier also has a random multiplier on the speed which ranges from zero to two, representing the wide variety of ski styles. After skiers reach the bottom, they rejoin the queue for the lifts, and their personal wait time timer restarts.

Using our NetLogo model, we found that the ideal characteristics for a lift was twenty-six lifts, each with a capacity of six people per chair, and having chairs that are spaced nine seconds apart and traveling at a speed of five meters/second. These inputs produced the graphs shown below.

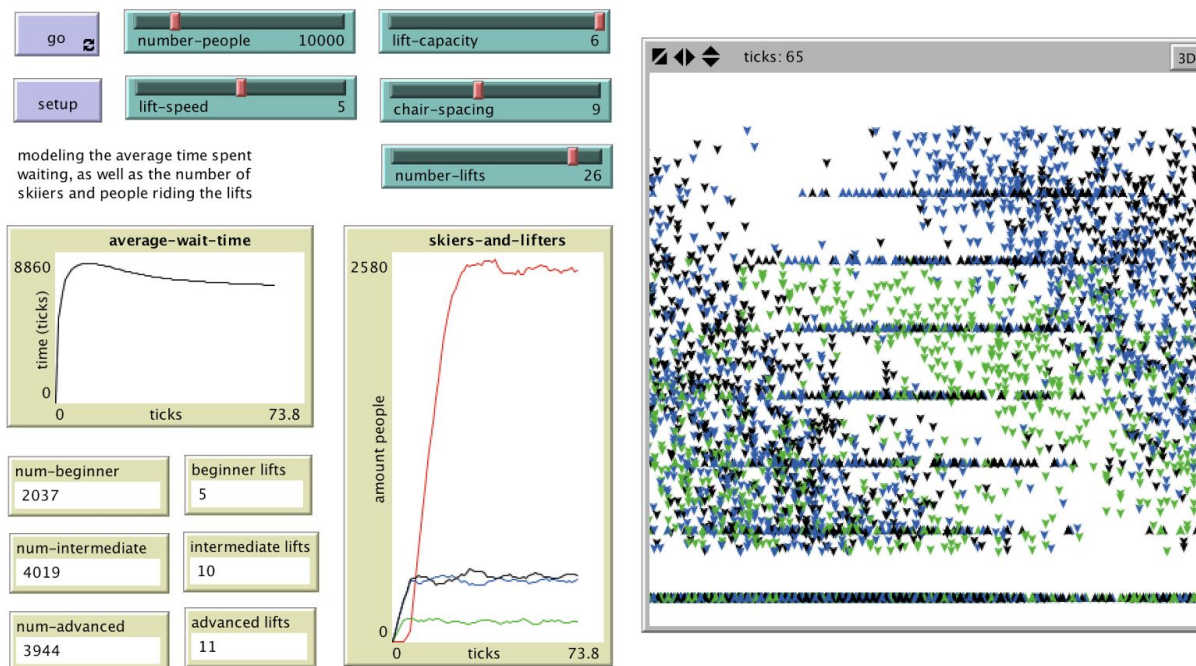


Figure 11: Graphs of Average Wait Time and the Number of Skiers and Lift Riders. The horizontal lines of triangles are the skiers riding the lift at the same speed. The triangles pointed downwards and at varying distances are skiers riding down the slope at different speeds.

From the graph of average wait time we see something interesting - the time rises quickly initially, but as time goes on it appears to decrease and approach an asymptote. This is due to the initial surge of people, and the time later stabilizes as the amount of people on the lifts and on the slopes reaches an equilibrium. The model is used to test results by analyzing the wait time and the number of skiers and lift riders. Our goal is to minimize the average wait time per person, while maximizing the ratio of people on the slopes to people on lifts and waiting. Changing the values using the sliders, such as increasing the lift speed or the chair spacing led to longer average wait times. This occurs because there exists a tradeoff between getting people onto slopes faster and having people wait at the bottoms. By increasing the amount of people on the slopes, we also increase turnover, as there will be more people coming down the slopes in the next iteration. Overall, the data obtained confirmed our choice of planned slope and lift layout for Wasatch Peaks Ranch. Our skier to rider ratio was 2.48, with 2.48 skiers for every person riding on a lift.

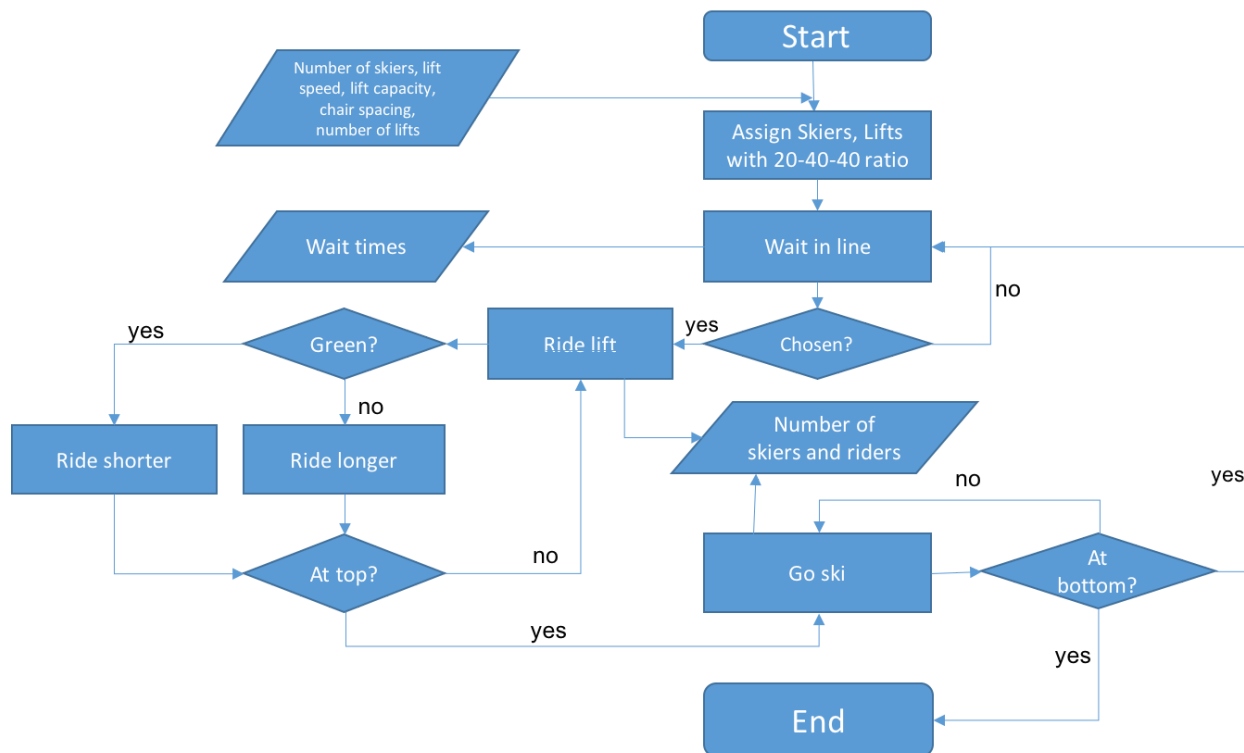


Figure 12: Flowchart of NetLogo program

Sensitivity Analysis

In order to test our model, we wanted to see how the rating of our design would change if we changed the number of lifts. Currently, it appears crowded, suggesting a reason to lower the amount of lifts; however, there are still specific stations that are disconnected, making a case for the addition of lifts. The calculations behind this analysis are rather simple. In our metric, only two parts are dependent on lifts - the walking space per lift (WSPL) and the number of slopes per lift (SPL). In the table below, we observed how the product of the metric values for each changes.

Lifts	WS	WSPL	$(1 - \frac{WSPL - WSPL^0}{WSPL^0})$	SPL	$(1 - \frac{SPL - SPL^0}{SPL^0})$	$(1 - \frac{WSPL - WSPL^0}{WSPL^0}) (1 - \frac{SPL - SPL^0}{SPL^0})$
11	5104.631	464.057	-0.392	14.545	0.678	-0.266
12	5104.631	425.386	-0.193	13.333	0.788	-0.152
13	5104.631	392.664	-0.024	12.308	0.881	-0.021
14	5104.631	364.617	0.121	11.429	0.961	0.116
15	5104.631	340.309	0.246	10.667	0.970	0.238
16	5104.631	319.039	0.355	10.000	0.909	0.323
17	5104.631	300.272	0.452	9.412	0.856	0.387
18	5104.631	283.591	0.538	8.889	0.808	0.435
19	5104.631	268.665	0.615	8.421	0.766	0.471
20	5104.631	255.232	0.684	8.000	0.727	0.498
21	5104.631	243.078	0.747	7.619	0.693	0.517
22	5104.631	232.029	0.804	7.273	0.661	0.532
23	5104.631	221.940	0.856	6.957	0.632	0.541
24	5104.631	212.693	0.904	6.667	0.606	0.548
25	5104.631	204.185	0.947	6.400	0.582	0.551
26	5104.631	196.332	0.988	6.154	0.559	0.553
27	5104.631	189.060	0.975	5.926	0.539	0.525
28	5104.631	182.308	0.940	5.714	0.519	0.488
29	5104.631	176.022	0.907	5.517	0.502	0.455
30	5104.631	170.154	0.877	5.333	0.485	0.425

31	5104.631	164.666	0.849	5.161	0.469	0.398
32	5104.631	159.520	0.822	5.000	0.455	0.374
33	5104.631	154.686	0.797	4.848	0.441	0.351

Figure 13: Sensitivity Analysis with Lifts. Highest value highlighted.

The table above justifies our decision to include twenty-six ski lifts. From this table, we can also conclude that 26 is the maximum we should include before problems are created as the value of this part of the metric begins to drop rapidly for each lift added after the 26th. On the contrary, a few could be removed with minimal effects.

Justification

We searched for alternate methods to evaluate robustness and stability of the model we developed using vector fields and morphological image analysis techniques. We turned to various other ways to analyze ski slopes. Our method of choice was an implementation of an agent-based watershed analysis of the Peterson county region using the SURFER program by Golden Software. As explained before, our research on the characteristics of popular ski slopes tells us that the best ski runs are fall-line pistes. The “fall line” describes the route an object would take down the slope if one let it roll from the top - a rough path of least resistance, so to speak. To evaluate that each of our trails indeed fell on “fall lines that already existed on the mountain, we ran the following watershed analysis:

To find the local minima altitudes in our graph, we split up our grid into basin, or catchment, areas. We colored basin areas as areas that drain water into streams, and calculated stream paths based on the amount of flow into the grid node from all surrounding grid nodes. To do this, we used SURFER’s built-in eight-direction pour point algorithm to calculate the flow direction at each node. Iterations through the model were conducted by manipulating the threshold value (in cells), the number of upstream grid nodes that must flow into a particular cell before a stream is created. We started at a large threshold value of 32000, meaning that more cells were required to drain into a cell for to create a stream and watershed basin. Consequently, fewer larger watershed basins were created. Further iterations decreased the threshold value, found more local minima, and extended previously existing minima. Naturally, more of the map was tested as we ran more simulations, and our sector colorings accordingly based on relative elevations. As can be seen in Figure 14, our method effectively follows the vector field quivers with greatest magnitude, building up from each local minimum to stop at each local maximum, revealing the fall lines within the mountain terrain.

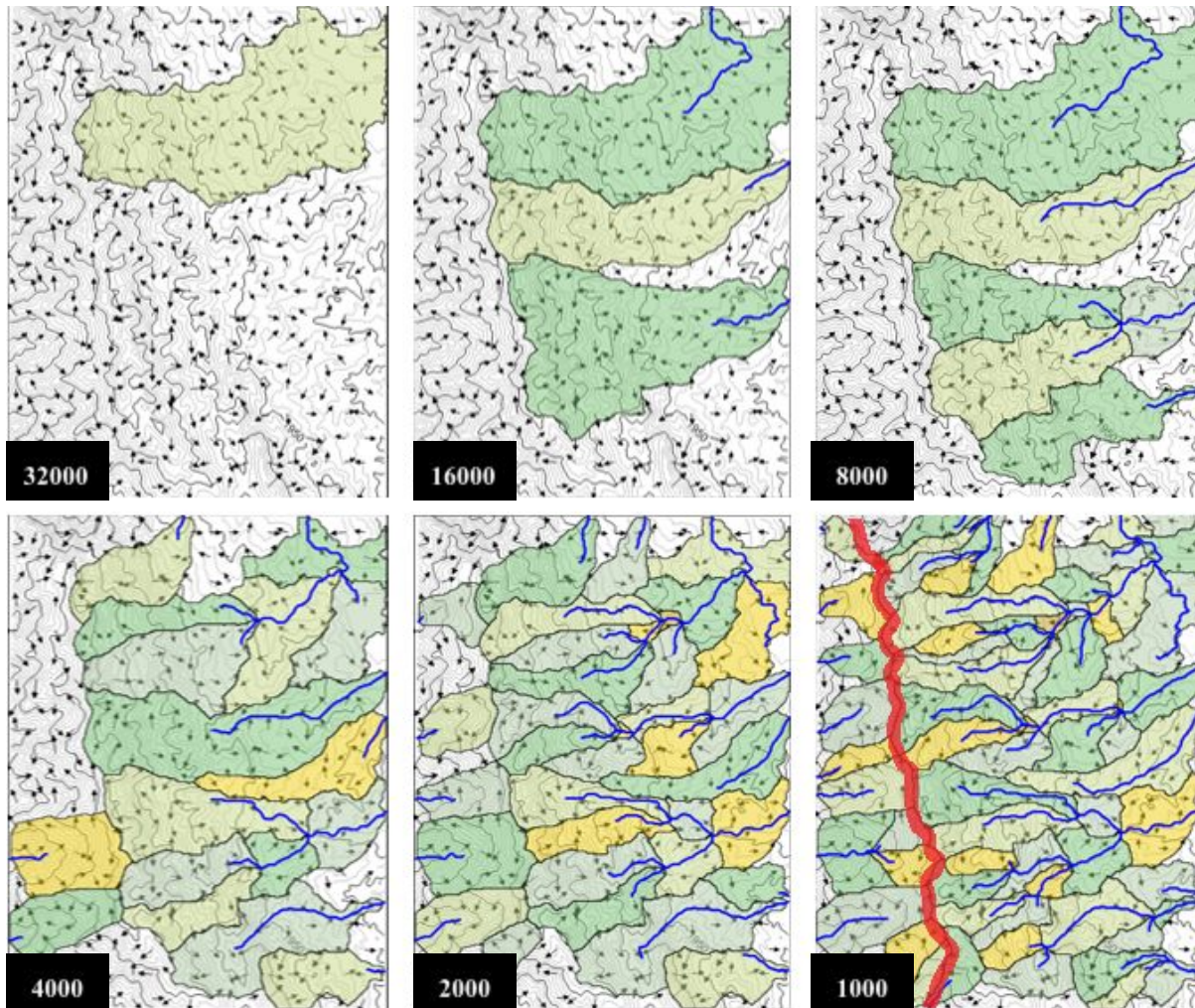


Figure 14: Watershed analysis iterations at representative threshold (cell) values overlaying contour and 2-Grid vector field plots of Peterson. T-values are written at the bottom left corner of each partition. The red line on the 1000-cell partition delineates the mountain range as sectioned by the basins. All of our ski slopes lie to the right of this range.

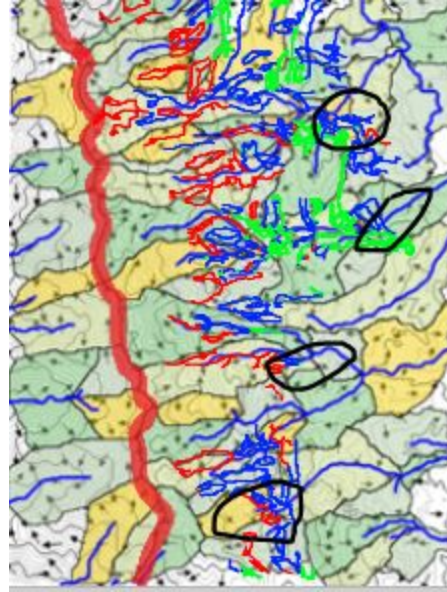


Figure 15: Comparison of SURFER-watershed (threshold value of 1000 cells) and Mathematica-vector field fall lines; the overlays show strong similarities between the two methods; a number of similar paths between watershed and vector field are circled to highlight this fact in support of our model's robustness.

After calculating the slope gradient, we created a north facing solar aspect map of the area. An overlay which highlighted these northern faces was placed over a topographical map using US Geological Survey data obtained from CalTopo [1]. Slopes facing from northwest to northeast were highlighted red, whereas slopes facing southeast to southwest were highlighted in a blue tint. Since Wasatch Peaks Ranch is located in the Northern Hemisphere, during winter times the angle and location of the sun causes south facing slopes to receive much more solar radiation. Solar radiation causes snow to melt and refreeze, leading to icy conditions which should be avoided. In our final model, slopes facing north are weighted more favorably for ski paths compared to south facing slopes. The figure below shows the aspect direction of the slopes, and has the Wasatch Peaks property overlaid.

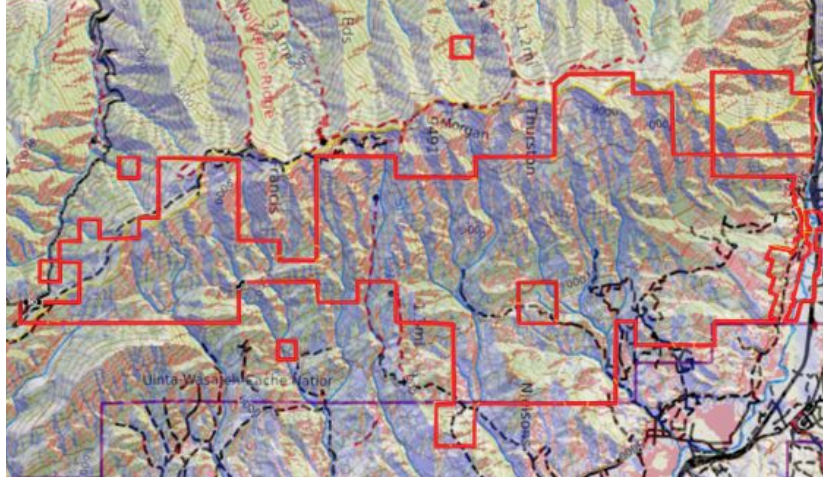


Figure 15: Solar Aspect Map with Wasatch Peaks Ranch overlaid in red

Strengths and Weaknesses

Strengths:

1. Our model combines three different methods of judging a slope for skiing: the slope gradient, the fall line, and the solar aspect. The combination of multiple methods ensures that we are taking more factors into account and that we don't overlook anything that may not appear in one model.
2. The computational model can be easily and quickly adapted to any other field, and is not specific to any specific ski resort. NetLogo is an interactive agent-based modeling program that allows it to be reused in the future by changing variables such as lift speed.
3. Our model accounts for stochastic randomness by incorporating random weightings into both the amount of people at any given lift as well as the speed at which people return to a lift. This supports the insensitivity of our model since the minor changes do not lead to any difference in results.
4. Our model allows for the testing and manipulation of many different variables, including the people in the park, the lift speed, the lift capacity, the chair spacing, and the number of lifts.
5. Our model is built to be environmentally friendly. We avoid streams and creeks for mutual safety and most of our slopes follow natural fall-lines. The entirety of our skiing resort is on natural terrain. This reduces costs of building (no major man-made alterations) and aligns with our company's and previous Winter Olympic venues' pillars of working with nature and letting its own beauty shine through.

Weaknesses:

1. Our model prioritizes following the fall line heavily, and does not consider options that could be created by flattening or bulldozing any paths parallel to contour lines.
2. Our model has erroneous data at the far northern and southern edges of the ranch due to flaws in GIS data.
3. Parts of our model were done manually due to computational difficulties; these may have resulted in slight inaccuracies lining up different layers of the solution or imperfect choices for final slope locations, in the step where we separated by difficulty.
4. Our model has slightly inaccurate gradients near the peaks of topographical features, as interpolations were done only within a distance of a data points from the origin due to the capabilities of Mathematica; this caused some slopes to deviate by one or two pixels from actual topography when projected onto our altitude features.
5. Our model does not account for relative safety differences between ridges and valleys, or any other desirable qualities in easy courses not desirable in difficult ones such as length or twistiness.
6. Our model does not take into account any differences between lift and ski slopes due to potential installation costs, e.g. on a flat plain versus on a steep slope.
7. Our model includes a few areas where lifts cross over property that is not part of the Wasatch Peaks Ranch. This is due to areas where the property narrows significantly, especially at its southern tip, where it measures a mere 1200 ft. across.

Conclusion

Using both mathematical and computational models, we evaluated potential ski slopes on the Wasatch Peaks Ranch property. We drew topological and altitude data from US Geological Survey databases, and used it to create a series of maps, including a slope gradient, solar aspect, and a vector field fall line. Overlaying the three graphs on top of each other allowed us to identify 199.4 km of ski slope, with a distribution of 22.72% green circle, 38.50% blue square, and 38.77% black diamond. Applying our ranking metric to the Wasatch Peaks Ranch indicates that it would rank as the fourth best ski resort out of the ones sampled in North America. However, our ski slope prides itself on being extremely ecologically friendly, using natural fall lines and preserving as much of the topography and landforms (rivers, basins, creeks, etc.) as possible, keeping costs low and showcasing the beauty of the American landscape for all to see at the 2026 Winter Olympics.

We also developed a computational agent-based model in NetLogo, which allows us to input a series of parameters and obtain an output graph detailing the average time spent waiting for lifts as well as the amount of people on the slopes compared to people riding the lifts. This analysis provides useful data for us to judge the amount of ski lifts required for a given park, with the

goal being to minimize time spent waiting and to maximize the ratio of people on the slopes to people on lifts.

Additional Extensions

Given more time and resources, we would pursue several interesting approaches further. Building off of our environmentally friendly approach to minimizing the amount of environmental modifications made, we would further classify slopes based on whether they occurred on a forested or a non-forested area. This data would be gathered from infrared imaging (see appendix figure A1). Minimizing the amount of trees removed would reduce erosion, preserving the snow paths for future years. Additionally, we would like to carry out more analysis of what makes an Olympic host Olympic-caliber. Olympic bids often have many factors that play a role, ranging from the security of the venue to the culture around it. Not only would we have to put effort into designing the best ski slopes, we would need to work on building up a community around it. This leads into another factor - villages. Many ski resorts have villages located within them, for ski-in ski-out travelers. Certainly, the factors that play a role in village housing placement are different than the ones that affect ski slope placement, and we would like to study these factors further. Lastly, with further time we would seek to perform a cost analysis evaluation, taking into account locations of ski lifts on a slope, and ease of grooming pistes.

References

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Appendix

Additional Figures -

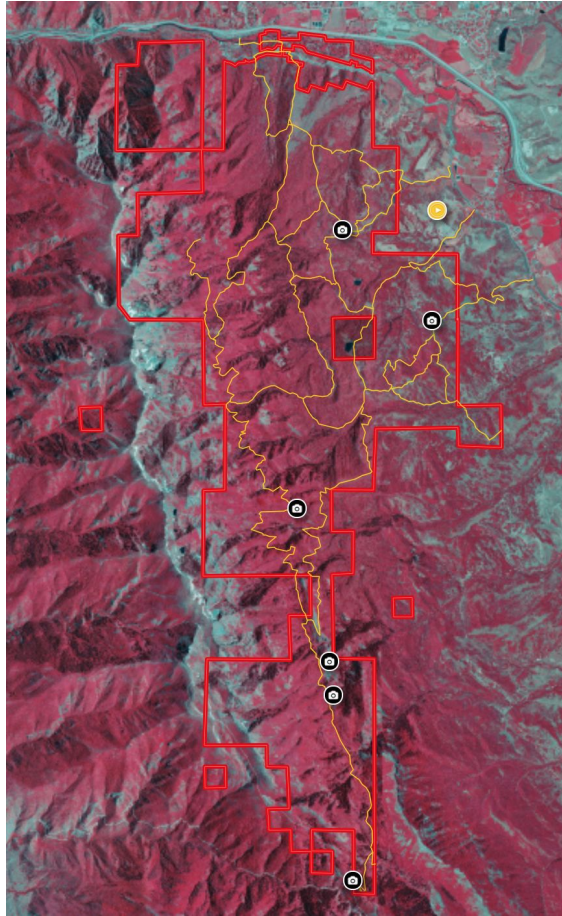


Figure A1: Infrared imaging of Peterson, UT, with Wasatch Peaks Ranch overlaid. Red colors indicate vegetation coverage.

NetLogo Code - NetLogo was used to model the average wait times and the numbers of skiers and lift riders of a park given certain input parameters. The code and blank interface are given below.



Figure A2: Blank Interface. Each skier sits at the base of one of twenty-six lifts.

```
globals [
  num-beginner
  num-intermediate
  num-advanced
  num-b-lift
  num-i-lift
  num-a-lift
  lift-velocity
  width
  time-waiting
]

turtles-own[
  speed
  wait-time
  skiing?
  lifting?
  get-out-of-the-way!
  get-in-line!
]

to setup
  clear-all
  ask patches [set pcolor white]
  create-turtles number-people
```

```
reset-ticks
setup-turtles
setup-lifts
beginner
intermediate
advanced
set time-waiting 0
end

to setup-turtles
  ask turtles [
    ifelse random-float 1 < .2 ;;randomly determining the number of people,
close to the 20/40/40 ratio of b/i/a
    [set color green]
    [ifelse random-float 1 < .5
      [set color blue]
      [set color black]
    ]
    setxy 20 0
    set heading 0
    set skiing? false
    set lifting? false
    set get-out-of-the-way! false
    set get-in-line! false
    set speed 0
    set wait-time 0
  ]
  set num-intermediate count turtles with [color = blue]
  set num-advanced count turtles with [color = black]
  set num-beginner count turtles with [color = green]
end

to setup-lifts
  set num-i-lift floor (number-lifts * 0.4) ;;allocating number of lifts to use
for the 3 difficulty levels
  set num-a-lift ceiling (number-lifts * 0.4)
  set num-b-lift (number-lifts - num-i-lift - num-a-lift)
  set width (40 / (number-lifts + 1))
  ask turtles [
    set ycor width
  ]
  set lift-velocity lift-speed
end

to beginner
  ask turtles with [color = green][ ;;beginner level, b
    ifelse random num-b-lift < 1
      [set xcor (width * 1)]
```

```
[ifelse random num-b-lift < 2
  [set xcor (width * 2)]
  [ifelse random num-b-lift < 3
    [set xcor (width * 3)]
    [ifelse random num-b-lift < 4
      [set xcor (width * 4)]
      [ifelse random num-b-lift < 5
        [set xcor (width * 5)]
        [set xcor (width * num-b-lift)]]
      ]
    ]
  ]
]
]
]
set speed 1
]

end

to intermediate
  ask turtles with [color = blue][ ;;intermediate level, i
    ifelse random num-i-lift < 1
      [set xcor (width * (num-b-lift + 1))] ;;drawing the intermediate ski lifts
      on the interface
      [ifelse random num-i-lift < 2
        [set xcor (width * (num-b-lift + 2))]
        [ifelse random num-i-lift < 3
          [set xcor (width * (num-b-lift + 3))]
          [ifelse random num-i-lift < 4
            [set xcor (width * (num-b-lift + 4))]
            [ifelse random num-i-lift < 5
              [set xcor (width * (num-b-lift + 5))]
              [ifelse random num-i-lift < 6
                [set xcor (width * (num-b-lift + 6))]
                [ifelse random num-i-lift < 7
                  [set xcor (width * (num-b-lift + 7))]
                  [ifelse random num-i-lift < 8
                    [set xcor (width * (num-b-lift + 8))]
                    [ifelse random num-i-lift < 9
                      [set xcor (width * (num-b-lift + 9))]
                      [ifelse random num-i-lift < 10
                        [set xcor (width * (num-b-lift + 10))]
                        [ifelse random num-i-lift < 11
                          [set xcor (width * (num-b-lift + 11))]
                          [set xcor (width * (num-b-lift + num-i-lift))]
                        ]
                      ]
                    ]
                  ]
                ]
              ]
            ]
          ]
        ]
      ]
    ]
  ]
]
]
]
```



```
end

to lift
  ask n-of (lift-capacity * number-lifts / (chair-spacing / 10) ) turtles with
  [skiing? = false and lifting? = false][
    set lifting? true ;;get on lift
  ]
  ask turtles with [lifting? = true][
    forward lift-velocity
  ]
  ask turtles with [color = green and ycor >= (30 - (lift-speed))][
    set lifting? false
    set skiing? true
    set get-out-of-the-way! true
    set get-in-line! false
  ]
  ask turtles with [ycor >= (40 - (lift-speed))][
    set lifting? false
    set skiing? true
    set get-out-of-the-way! true
    set get-in-line! false
  ]
end

to ski
  ask turtles with [get-out-of-the-way! = true][ ;;get off lift, ski back to
base
    set xcor (xcor + 2)
    set heading 180
    set get-out-of-the-way! false
  ]
  ask turtles with [skiing? = true][
    ifelse color = green
    [forward 1 * random-float 2]
    [ifelse color = blue
    [forward 2 * random-float 2]
    [forward 3 * random-float 2]
  ]
  ]
  ask turtles with [ycor <= (lift-speed)][
    set ycor width
    set lifting? false
    set skiing? false
    set get-in-line! true
  ]
end



to queue
```

```
ask turtles with [get-in-line! = true][ ;;get back in line for the lift
  set heading 0
  set xcor (xcor - 2)
  set get-in-line! false
]
set time-waiting time-waiting + (count turtles with [lifting? = false and
skiing? = false])
end

to go
  lift
  ski
  queue
  tick
end
```





```
ReliefPlot[intgrad3, ColorFunction -> "HypsometricTints",  
  ColorFunctionScaling -> False];  
orientgrad3 =  
  Image[GradientOrientationFilter[Reverse[data3], 2,  
    Method -> "ShenCastan"]] // ImageAdjust;  
  
list = ConformImages[{orientgrad1, orientgrad2, orientgrad3}];  
peterson = ImageAssemble[{{list[[1]]}, {list[[2]]},  
{list[[3]]}}]  
  
MorphologicalComponents[  
  DeleteSmallComponents[EdgeDetect[ImageResize[peterson,  
Scaled[1]]],  
  40]] // Colorize  
mean = {0, 0, 0};  
ImageScan[mean += # &,  
  i = DeleteSmallComponents[  
    EdgeDetect[ImageResize[peterson, Scaled[1]]], 40]]  
  
mean /= Times @@ ImageDimensions[i]  
(*this is the number of white pixels over the total pixels*)  
  
ListPlot3D[  
  Transpose[  
    Drop[Drop[  
      Transpose[  
        Drop[Drop[  
          Import["  
", "Data"], 5], -5]], 5], -5]],  
    PlotStyle ->  
      Texture[MorphologicalComponents[  
        DeleteSmallComponents[  
          EdgeDetect[ImageResize[list[[2]], Scaled[1]]], 40]] //  
          Colorize], Mesh -> 40, ImageSize -> Large,  
        PerformanceGoal -> "Quality", PlotTheme -> "Minimal"]
```

```
skislopes = ChanVeseBinarize[ImageCompose[
  DeleteSmallComponents[
    EdgeDetect[ImageResize[peterson, Scaled[1.15]]], 30],
  Import[
    "
"],
  {205, 810}
]];
Row[{ColorNegate[Image[skislopes, ImageSize -> 300]],
  Image[DeleteSmallComponents[
    EdgeDetect[ImageResize[peterson, Scaled[1]]], 30],
  ImageSize -> 300]}}
```

```
array // Colorize
```

```
For[w = 1, w < Length[array] + 1, w++,
  For[j = 1, j < Length[array[[1]]] + 1, j++,
    If[array[[w]][[j]] == 2, array[[w]][[j]] = 0]] (*erases
rivers manually*)
```

```
ColorNegate[ChanVeseBinarize[ImageCompose[
  DeleteSmallComponents[EdgeDetect[ImageResize[peterson,
Scaled[1]]],
  30],
  ImageResize[
    Import["
"], Scaled[0.87]],
  {178, 704}
]]]
```